

# The Cauchy problem of $f(R)$ gravity

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**Abstract.** The initial value problem of metric and Palatini  $f(R)$  gravity is studied by using the dynamical equivalence between these theories and Brans-Dicke gravity. The Cauchy problem is well-formulated for metric  $f(R)$  gravity in the presence of matter and well-posed in vacuo. For Palatini  $f(R)$  gravity, instead, the Cauchy problem is not well-formulated.

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## 1. Introduction

The study of type 1a supernovae led to the 1998 discovery of the acceleration of the cosmic expansion [1], which has prompted theoretical physicists to look for an explanation. Many models of this accelerated dynamics postulate the existence of dark energy, a diffuse mysterious form of energy with exotic equation of state  $P \approx -\rho$  which amounts to 70% of the critical energy density, with the remaining 30% comprised of dark and ordinary matter. As an alternative to exotic dark energy models, it has been proposed that infrared modifications of gravity at the largest scales could be the explanation of the cosmic acceleration [2]. The idea is either to consider the Einstein-Hilbert action<sup>‡</sup> with cosmological constant

$$S_{EH} = \frac{1}{2\kappa} \int d^4x \sqrt{-g} (R - 2\Lambda) + S^{(matter)} \quad (1.1)$$

or some form of dark energy mimicking  $\Lambda$ , or to change gravity as described by the alternative action

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g} f(R) + S^{(matter)}, \quad (1.2)$$

where  $f(R)$  is a non-linear function of  $R$ . This “ $f(R)$ ” or “modified” gravity has a long history (see Ref. [4] for an historical perspective): it was originally conceived as a mathematical alternative to Einstein’s theory [5]; quadratic corrections to the Einstein-Hilbert action were discovered to be necessary for the renormalization of general relativity [6], and then used to generate inflation in the early universe without scalar fields [7]; see Ref. [8] for a more modern perspective.

Modified gravity comes in three possible forms: the first is metric  $f(R)$  gravity, in which the metric is the only independent variable and the variation of the action (1.2) with respect to  $g^{ab}$  yields the fourth-order field equations

$$f'(R)R_{ab} - \frac{f(R)}{2}g_{ab} = \nabla_a \nabla_b f'(R) - g_{ab} \square f'(R) + \kappa T_{ab}^{(m)}. \quad (1.3)$$

The second possibility, named “Palatini  $f(R)$  gravity” consists of treating the metric and the connection as independent variables (*i.e.*, the connection  $\Gamma_{bc}^a$  is not the metric connection  $\{^a_{bc}\}$  of  $g_{ab}$ ). If the (non-metric) connection is assumed to be symmetric, the field equations (which are now of second order) are [9]

$$f'(\tilde{R})R_{ab} - \frac{f(\tilde{R})}{2}g_{ab} = \kappa T_{ab}^{(m)}, \quad (1.4)$$

$$-\tilde{\nabla}_c \left[ \sqrt{-g} f'(\tilde{R}) g^{ab} \right] + \tilde{\nabla}_d \left[ \sqrt{-g} f'(\tilde{R}) g^{d(a} \right] \delta_c^{b)} = 0, \quad (1.5)$$

where  $\tilde{R}_{ab}$  is the Ricci tensor of the non-metric connection,  $\tilde{R} = g^{ab} \tilde{R}_{ab}$ , and  $\tilde{\nabla}_c$  is the covariant derivative operator of  $\Gamma_{bc}^a$ .

<sup>‡</sup> Here  $\kappa \equiv 8\pi G$ ,  $R$  is the Ricci curvature,  $\Lambda$  is the cosmological constant, and  $g$  is the determinant of the metric tensor  $g_{ab}$ . We follow the notations of Ref. [3].

The third possibility, “metric-affine gravity”, allows also the matter action to depend on the independent connection, resulting in more general field equations [10]. In this paper, we only consider the metric and Palatini formalisms.

First, let us focus on metric  $f(R)$  gravity. In order for these theories to be viable, several criteria must be met, which have been analysed in the recent past. The theory must have the correct Newtonian and post-Newtonian limits [11, 12] and it must not have short timescale instabilities, which is achieved if  $f'' > 0$  [13, 14, 15, 16] (see Refs. [17] for studies of various instabilities). The theory must possess the correct cosmological dynamics, *i.e.*, an inflationary era, followed by a radiation era and then a matter era and, finally, an acceleration era. Furthermore, there must be smooth transitions between consecutive eras, which is not always achieved, with many models being ruled out [18]. The theory must also be ghost-free, which is the case for  $f(R)$  and for Gauss-Bonnet gravity, but not for theories of the form  $f(R, R_{ab}R^{ab}, R_{abcd}R^{abcd})$  [19]. Another criterion for validity that should certainly be satisfied is that the theory possesses a well-posed initial value formulation. Here we focus on that aspect. In addition to being a viability criterion, the Cauchy problem proves very useful in the numerical integration of the field equations.

The initial value problem was studied for theories of the form

$$S = \int d^4x \sqrt{-g} \left( \frac{R}{2\kappa} + \alpha R_{ab}R^{ab} + \beta R^2 + \gamma R_{abcd}R^{abcd} \right) + S^{matter} \quad (1.6)$$

in Refs. [20, 21] with the conclusion that the Cauchy problem is well-posed. Due to the Gauss-Bonnet identity in four spacetime dimensions, the Kretschmann scalar can be dropped from the action. In the following, we consider the Cauchy problem for the theory described by the action (1.2) and for a general form of the function  $f(R)$ , without restricting to the specific theory  $f(R) = R + \alpha R^2$  considered in Refs. [20, 21]. Instead of proceeding directly on the field equations (1.3), we will use a well known dynamical equivalence relation mapping  $f(R)$  into scalar-tensor theories [22, 21] and we will address the Cauchy problem in scalar-tensor gravity, relying on recent results on this subject [23]. Our main result is that the Cauchy problem for metric  $f(R)$  gravity is well-formulated, and is well-posed in vacuum. For Palatini  $f(R)$  gravity, instead, the Cauchy problem is not well-formulated nor well-posed due to the presence of higher derivatives of the matter fields in the field equations and to the impossibility of eliminating them. Here, the system of 3+1 equations of motion is said to be *well-formulated* if it can be re-written as a system of only first-order equations (in time and space) in the scalar field variables. When this set can be put in the full first order form

$$\partial_t \vec{u} + M^i \nabla_i \vec{u} = \vec{S}(\vec{u}) , \quad (1.7)$$

where  $\vec{u}$  collectively denotes the fundamental variables  $h_{ij}, K_{ij}$ , *etc.* introduced below,  $M^i$  is called the *characteristic matrix* of the system, and  $\vec{S}(\vec{u})$  describes source terms and contains only the fundamental variables but not their derivatives. Then, the initial value formulation is *well-posed* if the system of partial differential equations is *symmetric hyperbolic* (*i.e.*,  $M^i$  are symmetric) and *strongly hyperbolic* (*i.e.*,  $s_i M^i$  has a real set

of eigenvalues and a complete set of eigenvectors for any 1-form  $s_i$ , and obeys some boundedness conditions). We refer the reader to Ref. [24] for more precise definitions and for a complete discussion.

The plan of this paper is the following: in Sec. 2 we recall the equivalence between  $f(R)$  and scalar-tensor gravity. In Sec. 3 we proceed to show that the Cauchy problem is well-formulated for the scalar-tensor equivalent of metric  $f(R)$  gravity. The scalar-tensor equivalent of Palatini  $f(R)$  gravity is considered in Sec. 4, and Sec. 5 contains a discussion and the conclusions.

## 2. The equivalence between $f(R)$ and scalar-tensor gravity

We briefly review the equivalence between  $f(R)$  and scalar-tensor gravity, starting with the metric formalism. By introducing the extra scalar field  $\phi$ , the modified gravity action

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g} f(R) + S^{(matter)} \quad (2.1)$$

can be rewritten as [22, 21]

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g} [\psi(\phi)R - V(\phi)] + S^{(matter)} \quad (2.2)$$

when  $f''(R) \neq 0$ , where

$$\psi = f'(\phi), \quad V(\phi) = \phi f'(\phi) - f(\phi). \quad (2.3)$$

The action (2.2) coincides with (2.1) if  $\phi = R$ . Vice-versa, the variation of (2.2) with respect to  $\phi$  yields

$$R \frac{d\psi}{d\phi} - \frac{dV}{d\phi} = (R - \phi) f''(R) = 0, \quad (2.4)$$

which has no dynamical content but implies<sup>§</sup> that  $\phi = R$  when  $f''(R) \neq 0$ . (2.2) is a Brans-Dicke action of the form [26]

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g} \left[ \psi R - \frac{\omega}{2} \nabla^c \psi \nabla_c \psi - U(\psi) \right] + S^{(matter)} \quad (2.5)$$

with Brans-Dicke field  $\psi$ ,  $U(\psi) = V[\phi(\psi)]$ , and Brans-Dicke parameter  $\omega = 0$ . This is called O'Hanlon theory or “massive dilaton gravity” and was originally introduced in order to produce a Yukawa term in the Newtonian potential when taking the Newtonian limit [27]. The field equations are

$$G_{ab} = \frac{\kappa}{\psi} T_{ab}^{(m)} - \frac{1}{2\psi} U(\psi) g_{ab} + \frac{1}{\psi} (\nabla_a \nabla_b \psi - g_{ab} \square \psi), \quad (2.6)$$

$$3\square\psi + 2U(\psi) - \psi \frac{dU}{d\psi} = \kappa T^{(m)}. \quad (2.7)$$

Let us consider now modified gravity in the Palatini formalism. The Palatini action

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g} f(\tilde{R}) + S^{(matter)} \quad (2.8)$$

<sup>§</sup> An action of the form (2.1) with the property that  $f''(R) \neq 0$  is sometimes called “ $R$ -regular” [25].

is equivalent to

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g} \left[ f(\chi) + f'(\chi) (\tilde{R} - \chi) \right] + S^{(matter)}, \quad (2.9)$$

the variation of which with respect to  $\chi$  yields  $\chi = \tilde{R}$ . One now uses the field  $\phi = f'(\chi)$  and takes advantage of the fact that the curvature  $\tilde{R}$  can be seen as the Ricci curvature associated to the new metric

$$h_{ab} = f'(\tilde{R}) g_{ab} \quad (2.10)$$

conformal to  $g_{ab}$  [28, 10]. The two conformal frames are physically equivalent (see the recent extensive discussion of [29] and references therein). The transformation property of the Ricci scalar under conformal transformations is [30, 3]

$$\tilde{R} = R + \frac{3}{2\phi} \nabla^c \phi \nabla_c \phi - \frac{3}{2} \square \phi, \quad (2.11)$$

which allows the action (2.9) to be rewritten as

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g} \left[ \phi R + \frac{3}{2\phi} \nabla^c \phi \nabla_c \phi - V(\phi) \right] + S^{(matter)}, \quad (2.12)$$

where an irrelevant boundary term has been omitted and

$$V(\phi) = \phi \chi(\phi) - f[\chi(\phi)]. \quad (2.13)$$

The action (2.12) describes a Brans-Dicke theory with parameter  $\omega = -3/2$ . This theory is generally regarded as a pathological case [31], but is sometimes studied [32].

A general Brans-Dicke theory

$$S_{BD} = \frac{1}{2\kappa} \int d^4x \sqrt{-g} \left[ \phi R - \frac{\omega}{\phi} \nabla^c \phi \nabla_c \phi - V(\phi) \right] + S^{(matter)} \quad (2.14)$$

leads to an effective gravitational coupling appearing in a Cavendish experiment (and in the theory of cosmological perturbations [33])

$$G_{eff} = \frac{2(\omega + 2)}{(2\omega + 3)} \frac{1}{\phi}, \quad (2.15)$$

which becomes ill defined for  $\omega = -3/2$ . Similarly, the equation of motion for  $\phi$ ,

$$(2\omega + 3) \square \phi = \kappa T^{(m)} + \phi \frac{dV}{d\phi} - 2V(\phi) \quad (2.16)$$

reduces to an algebraic identity when  $\omega = -3/2$  and the field  $\phi$  becomes non-dynamical. This has obviously consequences for the initial value problem of the theory, which are discussed later.

Our strategy will be to study the Cauchy problem of  $f(R)$  gravity by reducing it to the initial value problem of  $\omega = 0, -3/2$  Brans-Dicke theory. Nevertheless, we start by considering more general scalar-tensor theories in the next section.

### 3. The initial value problem of scalar-tensor gravity

The Cauchy problem of Brans-Dicke and scalar-tensor gravity theories was considered in Refs. [20, 21, 34]. Noakes [20] showed that the Cauchy problem is well-posed for a non-minimally coupled scalar field  $\phi$  with action

$$S = \int d^4x \sqrt{-g} \left[ \left( \frac{1}{2\kappa} - \xi\phi^2 \right) R - \frac{1}{2} \nabla^c \phi \nabla_c \phi - V(\phi) \right] \quad (3.1)$$

without matter. Originally, this theory was introduced by regarding  $\phi$  as a form of (quantum) matter on a fixed curved background, not as a gravitational scalar field akin to the Brans-Dicke field, nor as a source of gravity on the right hand side of the Einstein equations. There is little doubt that this is the reason why Noakes did not include “ordinary” matter in the theory, which would complicate the discussion and weaken his conclusion of well-posedness of the Cauchy problem  $\parallel$ . However, (3.1) can be legitimately regarded as a scalar-tensor action.

Cocke and Cohen [34] sketched a study using Gaussian normal coordinates for Brans-Dicke theory with a free Brans-Dicke scalar. A covariant and systematic approach to the Cauchy problem of scalar-tensor theories of the form

$$S = \int d^4x \sqrt{-g} \left[ \frac{f(\phi)R}{2\kappa} - \frac{1}{2} \nabla^c \phi \nabla_c \phi - V(\phi) \right] + S^{(matter)} \quad (3.2)$$

has recently been advanced by Salgado [23], who showed that the initial value problem for these theories is well-posed in the absence of matter and well-formulated otherwise. For our purposes it is necessary to generalize Salgado’s results to more general scalar-tensor theories of the form

$$S = \int d^4x \sqrt{-g} \left[ \frac{f(\phi)R}{2\kappa} - \frac{\omega(\phi)}{2} \nabla^c \phi \nabla_c \phi - V(\phi) \right] + S^{(matter)} , \quad (3.3)$$

which contain the additional coupling function  $\omega(\phi)$ . In the following, we show in detail how this generalization proceeds and then specialize the discussion to the cases  $\omega = 0, -3/2$  which are the main focus of our interest. We follow as closely as possible the notations of Ref. [23] in order to facilitate comparison.

The field equations are (we set  $\kappa = 1$  in the rest of this paper)

$$\begin{aligned} G_{ab} = & \frac{1}{f} [f'' (\nabla_a \phi \nabla_b \phi - g_{ab} \nabla^c \phi \nabla_c \phi) + f' (\nabla_a \nabla_b \phi - g_{ab} \square \phi)] \\ & + \frac{1}{f} \left[ \omega \left( \nabla_a \phi \nabla_b \phi - \frac{1}{2} g_{ab} \nabla^c \phi \nabla_c \phi \right) - V(\phi) g_{ab} + T_{ab}^{(m)} \right] , \end{aligned} \quad (3.4)$$

$$\omega \square \phi + \frac{f'}{2} R - V'(\phi) + \frac{\omega'}{2} \nabla^c \phi \nabla_c \phi = 0 , \quad (3.5)$$

$\parallel$  This example shows that there is no clear-cut distinction between “gravitational” and “non-gravitational” scalar fields in relativistic theories of gravity (see Ref. [35] for a discussion and implications).

where a prime denotes differentiation with respect to  $\phi$ . Eq. (3.4) is rewritten as the effective Einstein equation

$$G_{ab} = T_{ab}^{(eff)} , \quad (3.6)$$

where the effective stress-energy tensor is [23]

$$T_{ab}^{(eff)} = \frac{1}{f(\phi)} \left( T_{ab}^{(f)} + T_{ab}^{(\phi)} + T_{ab}^{(m)} \right) , \quad (3.7)$$

with

$$T_{ab}^{(f)} = f''(\phi) (\nabla_a \phi \nabla_b \phi - g_{ab} \nabla^c \phi \nabla_c \phi) + f'(\phi) (\nabla_a \nabla_b \phi - g_{ab} \square \phi) , \quad (3.8)$$

which contains second order derivatives of  $\phi$  and is identical to the one in Ref. [23], and

$$T_{ab}^{(\phi)} = \omega(\phi) \left( \nabla_a \phi \nabla_b \phi - \frac{1}{2} g_{ab} \nabla^c \phi \nabla_c \phi \right) - V(\phi) g_{ab} \quad (3.9)$$

which has canonical structure (*i.e.*, quadratic in the first-order derivatives of  $\phi$  with no second order derivatives) and differs from the one of Ref. [23] by the presence of the coupling function  $\omega(\phi)$ . By taking the trace of eq. (3.6) and solving for  $\square \phi$ , one obtains

$$\square \phi = \frac{\frac{f' T^{(m)}}{2} - 2f' V(\phi) + f V'(\phi) + \left[ -\frac{\omega' f}{2} - \frac{f'}{2} (\omega + 3f'') \right] \nabla^c \phi \nabla_c \phi}{f \left[ \omega + \frac{3(f')^2}{2f} \right]} . \quad (3.10)$$

The 3+1 ADM formulation of the theory proceeds by defining the usual quantities lapse, shift, extrinsic curvature, and gradients of  $\phi$  [3, 36, 23]. Assume that a time function  $t$  exists such that the spacetime  $(M, g_{ab})$  can be foliated by a family of hypersurfaces  $\Sigma_t$  of constant  $t$  with unit timelike normal  $n^a$ . The 3-metric is defined by  $h_{ab} = g_{ab} + n_a n_b$  and  $h^a_c$  is the projection operator on  $\Sigma_t$ , with

$$n^a n_a = -1 , \quad h_{ab} n^b = h_{ab} n^a = 0 , \quad h_a^b h_{bc} = h_{ac} . \quad (3.11)$$

The metric is written using the lapse function  $N$  and the shift vector  $N^a$  as

$$ds^2 = - (N^2 - N^i N_i) dt^2 - 2N_i dt dx^i + h_{ij} dx^i dx^j \quad (3.12)$$

( $i, j = 1, 2, 3$ ), with  $N > 0$ ,  $n_a = -N \nabla_a t$  and

$$N^a = -h^a_b t^b , \quad (3.13)$$

where  $t^a$  is a time flow vector satisfying  $t^a \nabla_a t = 1$  and

$$t^a = -N^a + N n^a \quad (3.14)$$

so that  $N = -n_a t^a$  and  $N^a n_a = 0$ . The extrinsic curvature of  $\Sigma_t$  is

$$K_{ab} = -h_a^c h_b^d \nabla_c n_d \quad (3.15)$$

and the 3D covariant derivative of  $h_{ab}$  on  $\Sigma_t$  is defined by

$$D_i^{(3)} T^{a_1 \dots b_1 \dots} = h^{a_1}_{c_1} \dots h^{d_1}_{b_1} \dots h^f_i \nabla_f^{(3)} T^{c_1 \dots d_1 \dots} \quad (3.16)$$

for any 3-tensor  $^{(3)}T^{a_1 \dots b_1 \dots}$ , with  $D_i h_{ab} = 0$ . The spatial gradient of the scalar field is

$$Q_a \equiv D_a \phi , \quad (3.17)$$

while its momentum is

$$\Pi = \mathcal{L}_n \phi = n^c \nabla_c \phi \quad (3.18)$$

with

$$K_{ij} = -\nabla_i n_j = -\frac{1}{2N} \left( \frac{\partial h_{ij}}{\partial t} + D_i N_j + D_j N_i \right) , \quad (3.19)$$

$$\Pi = \frac{1}{N} (\partial_t \phi + N^c Q_c) , \quad (3.20)$$

and

$$\partial_t Q_i + N^l \partial_l Q_i + Q_l \partial_i N^l = D_i (N \Pi) . \quad (3.21)$$

The  $3+1$  decomposition of the effective stress-energy tensor  $T_{ab}^{(eff)}$  follows:

$$T_{ab}^{(eff)} = \frac{1}{f} (S_{ab} + J_a n_b + J_b n_a + E n_a n_b) , \quad (3.22)$$

where

$$S_{ab} \equiv h_a^c h_b^d T_{cd}^{(eff)} = \frac{1}{f} \left( S_{ab}^{(f)} + S_{ab}^{(\phi)} + S_{ab}^{(m)} \right) , \quad (3.23)$$

$$J_a \equiv -h_a^c T_{cd}^{(eff)} n^d = \frac{1}{f} \left( J_a^{(f)} + J_a^{(\phi)} + J_a^{(m)} \right) , \quad (3.24)$$

$$E \equiv n^a n^b T_{ab}^{(eff)} = \frac{1}{f} \left( E^{(f)} + E^{(\phi)} + E^{(m)} \right) , \quad (3.25)$$

and  $T^{(eff)} = S - E$ , where  $T^{(eff)} \equiv T^{(eff)a}_a$  and  $S \equiv S^a_a$ . By means of the Gauss-Codacci equations, the effective Einstein equations projected tangentially and orthogonally to  $\Sigma_t$  provide the Hamiltonian constraint [3, 23]

$$^{(3)}R + K^2 - K_{ij} K^{ij} = 2E , \quad (3.26)$$

the vector (or momentum) constraint

$$D_l K^l_i - D_i K = J_i , \quad (3.27)$$

and the dynamical equations

$$\begin{aligned} \partial_t K^i_j + N^l \partial_l K^i_j + K^i_l \partial_j N^l - K^l_j \partial_l N^i + D^i D_j N \\ - ^{(3)}R^i_j N - N K K^i_j = \frac{N}{2} [(S - E) \delta_j^i - 2S_j^i] , \end{aligned} \quad (3.28)$$

where  $K \equiv K^i_i$ . The trace of this equation yields

$$\partial_t K + N^l \partial_l K + ^{(3)}\Delta N - N K_{ij} K^{ij} = \frac{N}{2} (S + E) , \quad (3.29)$$

where  $^{(3)}\Delta \equiv D^i D_i$ . As remarked in [20, 23], the presence of second order derivatives of the scalar  $\phi$  (seen here as an effective form of “matter”) could potentially render the Cauchy problem ill-formulated. However, this is not the case because these derivatives can be eliminated in most cases (with one notable exception explained later) [23].



The  $f$ - and  $\phi$ -quantities appearing in eqs. (3.23)-(3.25) are then calculated, yielding

$$E^{(f)} = f' (D^c Q_c + K\Pi) + f'' Q^2 , \quad (3.30)$$

$$J_a^{(f)} = -f' (K_a^c Q_c + D_a \Pi) - f'' \Pi Q_a , \quad (3.31)$$

$$S_{ab}^{(f)} = f' (D_a Q_b + \Pi K_{ab} - h_{ab} \square \phi) - f'' [h_{ab} (Q^2 - \Pi^2) - Q_a Q_b] , \quad (3.32)$$

where  $Q^2 \equiv Q^c Q_c$ . Other useful quantities are

$$S^{(f)} = f' (D_c Q^c + K\Pi - 3\square \phi) + f'' (3\Pi^2 - 2Q^2) , \quad (3.33)$$

$$S^{(f)} - E^{(f)} = -3f' \square \phi - 3f'' (Q^2 - \Pi^2) . \quad (3.34)$$

All these  $f$ -quantities coincide with those of Ref. [23]. However, the following  $\phi$ -quantities differ from the corresponding ones of [23] by the presence of terms in  $\omega$  and  $\omega'$ :

$$E^{(\phi)} = \frac{\omega}{2} (\Pi^2 + Q^2) + V(\phi) , \quad (3.35)$$

$$J_a^{(\phi)} = -\omega \Pi Q_a , \quad (3.36)$$

$$S_{ab}^{(\phi)} = \omega Q_a Q_b - h_{ab} \left[ \frac{\omega}{2} (Q^2 - \Pi^2) + V(\phi) \right] , \quad (3.37)$$

while

$$S^{(\phi)} = \frac{\omega}{2} (3\Pi^2 - Q^2) - 3V(\phi) \quad (3.38)$$

and

$$S^{(\phi)} - E^{(\phi)} = \omega (\Pi^2 - Q^2) - 4V(\phi) . \quad (3.39)$$

The “total” quantities entering the right hand side of the 3 + 1 field equations are then

$$E = \frac{1}{f} \left[ f' (D^c Q_c + K\Pi) + \left( f'' + \frac{\omega}{2} \right) Q^2 + \frac{\omega}{2} \Pi^2 + V(\phi) + E^{(m)} \right] , \quad (3.40)$$

$$J_a = \frac{1}{f} \left[ -f' (K_a^c Q_c + D_a \Pi) - (f'' + \omega) \Pi Q_a + J_a^{(m)} \right] , \quad (3.41)$$

$$\begin{aligned} S_{ab} = & \frac{1}{f} \left\{ f' (D_a Q_b + \Pi K_{ab}) - h_{ab} \left[ \left( f'' + \frac{\omega}{2} \right) (Q^2 - \Pi^2) + V(\phi) + f' \square \phi \right] \right\} \\ & + \frac{1}{f} \left[ (\omega + f'') Q_a Q_b + S_{ab}^{(m)} \right] , \end{aligned} \quad (3.42)$$

while

$$\begin{aligned} S = & \frac{1}{f} [f' (D^c Q_c + \Pi K) - 3V(\phi) - 3f' \square \phi] \\ & - \frac{1}{f} \left[ \left( 2f'' + \frac{\omega}{2} \right) Q^2 - 3 \left( f'' + \frac{\omega}{2} \right) \Pi^2 + S^{(m)} \right] , \end{aligned} \quad (3.43)$$

$$S - E =$$

$$\frac{1}{f} [(3f'' + \omega) (\Pi^2 - Q^2) - 4V(\phi) - 3f' \square \phi + S^{(m)} - E^{(m)}] , \quad (3.44)$$

$$\begin{aligned} S + E &= \frac{1}{f} [2f' (D^c Q_c + K\Pi) - f'' Q^2 + (3f'' + 2\omega) \Pi^2] \\ &+ \frac{1}{f} (-2V(\phi) - 3f' \square \phi + S^{(m)} + E^{(m)}) . \end{aligned} \quad (3.45)$$

The Hamiltonian constraint becomes

$$\begin{aligned} {}^{(3)}R + K^2 - K_{ij} K^{ij} - \frac{2}{f} \left[ f' (D_c Q^c + K\Pi) + \frac{\omega}{2} \Pi^2 + \frac{Q^2}{2} (\omega + 2f'') \right] \\ = \frac{2}{f} (E^{(m)} + V(\phi)) , \end{aligned} \quad (3.46)$$

the momentum constraint (3.27) is

$$D_l K^l_i - D_i K + \frac{1}{f} [f' (K_i^c Q_c + D_i \Pi) + (\omega + f'') \Pi Q_i] = \frac{J_i^{(m)}}{f} , \quad (3.47)$$

the dynamical equation (3.28) is written as

$$\begin{aligned} \partial_t K^i_j + N^l \partial_l K^i_j + K^i_l \partial_j N^l - K_j^l \partial_l N^i + D^i D_j N - {}^{(3)}R^i_j N - N K K^i_j \\ + \frac{N}{2f} [f'' (Q^2 - \Pi^2) + 2V(\phi) + f' \square \phi] \delta_j^i + \frac{N f'}{f} (D^i Q_j + \Pi K^i_j) \\ + \frac{N}{f} (\omega + f'') Q^i Q_j = \frac{N}{2f} [(S^{(m)} - E^{(m)}) \delta_j^i - 2S^{(m) i}{}_j] , \end{aligned} \quad (3.48)$$

with trace

$$\begin{aligned} \partial_t K + N^l \partial_l K + {}^{(3)}\Delta N - N K_{ij} K^{ij} - \frac{N f'}{f} (D^c Q_c + \Pi K) \\ + \frac{N}{2f} [f'' Q^2 - (2\omega + 3f'') \Pi^2] = \frac{N}{2f} (-2V(\phi) - 3f' \square \phi + S^{(m)} + E^{(m)}) \end{aligned} \quad (3.49)$$

where (cf. Ref. [23])

$$\begin{aligned} \mathcal{L}_n \Pi - \Pi K - Q^c D_c (\ln N) - D_c Q^c &= -\square \phi \\ &= -\frac{1}{f \left[ \omega + \frac{3(f')^2}{2f} \right]} \left( \frac{f' T^{(m)}}{2} - 2f' V(\phi) + f V'(\phi) \right) \\ &- \frac{1}{f \left[ \omega + \frac{3(f')^2}{2f} \right]} \left\{ \left[ \frac{-\omega' f}{2} - (\omega + 3f'') \frac{f'}{2} \right] \nabla^c \phi \nabla_c \phi \right\} . \end{aligned} \quad (3.50)$$

In vacuo, the initial data  $(h_{ij}, K_{ij}, \phi, Q_i, \Pi)$  on an initial hypersurface  $\Sigma_0$  must satisfy the constraints (3.46) and (3.47) plus

$$Q_i - D_i \phi = 0 , \quad (3.51)$$

$$D_i Q_j = D_j Q_i . \quad (3.52)$$

In the presence of matter, the variables  $E^{(m)}$ ,  $J_a^{(m)}$ ,  $S_{ab}^{(m)}$  must also be specified on the initial hypersurface. Gauge-fixing is equivalent to assigning lapse  $N$  and shift  $N^a$  (various gauge conditions used in numerical analysis are reviewed in [23]). The system (3.46)-(3.49) contains only first-order derivatives in both space and time once the d'Alembertian  $\square\phi$  is written in terms of  $\phi$ ,  $\nabla^c\phi\nabla_c\phi$ ,  $f$ , and its derivatives by using eq. (3.50). This system differs from the corresponding one in [23] only by terms in  $\omega(\phi)$  and its first derivative. From now on, everything will proceed as in Ref. [23] (with one exception discussed in the next section). The reduction to a first-order system indicates that the Cauchy problem is well-posed in vacuo, this time also for the more general scalar-tensor theories containing the extra coupling function  $\omega(\phi)$ , and that it is well-formulated in the presence of matter. We do not repeat Salgado's analysis here, referring the reader to [23]. We are now ready to translate the results in terms of  $f(R)$  gravity.

#### 4. The Cauchy problem of modified gravity

In the notations of Ref. [23] that we adopted, Brans-Dicke theory [26] is described by  $\omega(\phi) = \omega_0/\phi$  with  $\omega_0$  the constant Brans-Dicke parameter,  $f(\phi) = \phi$ , and  $V \rightarrow 2V$ . This yields the constraints

$$\begin{aligned} & {}^{(3)}R + K^2 - K_{ij}K^{ij} - \frac{2}{\phi} \left[ D^c Q_c + K\Pi + \frac{\omega_0}{2\phi} (\Pi^2 + Q^2) \right] \\ &= \frac{2}{\phi} [E^{(m)} + V(\phi)] , \end{aligned} \quad (4.1)$$

$$D_l K^l_i - D_i K + \frac{1}{\phi} \left( K_i^l Q_l + D_i \Pi + \frac{\omega_0}{\phi} \Pi Q_i \right) = \frac{J_i^{(m)}}{\phi} , \quad (4.2)$$

and the dynamical equations

$$\begin{aligned} & \partial_t K^i_j + N^l \partial_l K^i_j + K^i_l \partial_j N^l - K_j^l \partial_l N^i + D^i D_j N \\ & - {}^{(3)}R^i_j N - N K K^i_j + \frac{N}{2\phi} \delta_j^i (2V(\phi) + \square\phi) + \frac{N}{\phi} (D^i Q_j + \Pi K^i_j) \\ & + \frac{N\omega_0}{\phi^2} Q^i Q_j = \frac{N}{2\phi} ((S^{(m)} - E^{(m)}) \delta_j^i - 2S^{(m) i}{}_j) , \end{aligned} \quad (4.3)$$

$$\begin{aligned} & \partial_t K + N^l \partial_l K + {}^{(3)}\Delta N - N K_{ij} K^{ij} - \frac{N}{\phi} (D^c Q_c + \Pi K) - \frac{\omega_0 N}{\phi^2} \Pi^2 \\ &= \frac{N}{2\phi} [-2V(\phi) - 3\square\phi + S^{(m)} + E^{(m)}] , \end{aligned} \quad (4.4)$$

with

$$\left( \omega_0 + \frac{3}{2} \right) \square\phi = \frac{T^{(m)}}{2} - 2V(\phi) + \phi V'(\phi) + \frac{\omega_0}{\phi} (\Pi^2 - Q^2) . \quad (4.5)$$

>From the discussion of the previous sections, we conclude that metric  $f(R)$  gravity, which is equivalent to  $\omega_0 = 0$  Brans-Dicke gravity, has a well-formulated Cauchy problem in general and is also well-posed in vacuo. On the other hand, Palatini  $f(R)$  gravity, which is equivalent to a Brans-Dicke theory with  $\omega_0 = -3/2$ , is in trouble. In fact, for this value of the Brans-Dicke parameter, the d'Alembertian  $\square\phi$  disappears from eq. (4.5) and the field  $\phi$  is not dynamical—it can be arbitrarily assigned on a region or on the entire spacetime, provided its gradient satisfies the degenerate equation (4.5), which reduces to a constraint.

As a consequence, in the Palatini version, it is impossible to eliminate  $\square\phi$  from the system (4.1)-(4.4) unless  $\square\phi = 0$ . This condition includes both the possibility  $\phi = \text{constant}$ , in which case the theory degenerates to general relativity, and the case of a harmonic  $\phi$ , which is associated to a null  $\phi$ -wave. Apart from these special cases, the previous considerations constitute a no-go theorem for Palatini  $f(R)$  gravity, which has an ill-formulated Cauchy problem in vacuo and, therefore, can hardly be regarded as a viable theory. The absence of a well-posed Cauchy problem was briefly noticed in Ref. [20].

## 5. Discussion and conclusions

While metric  $f(R)$  gravity has a well-formulated initial value problem (and a well-posed one in vacuo), Palatini modified gravity does not. In conjunction with recent criticism of the Palatini formalism (according to which the correct Palatini variation necessitates a Lagrange multiplier method which is usually neglected and, when taken into account, produces essentially the same theory as the metric version [38]—see also [39]), this fact probably leads to the demise of Palatini  $f(R)$  gravity as a legitimate model of the cosmic acceleration. However, something can be learned from the situation of the Palatini version of modified gravity. It is interesting to speculate on the possible physical consequences of the failure of a theory to have a well-posed initial value problem for the reasons discussed in the previous section. As seen above, the difficulty originates from the fact that the effective “matter”, *i.e.*, the scalar field  $\phi$ , is described by a stress-energy tensor containing derivatives of  $\phi$  of second order instead of first. This non-canonical form makes it very difficult, if not impossible, to satisfy the energy conditions [3]. This feature in itself should not necessarily be regarded as a curse or failure. However, while for the  $\omega = 0$  equivalent of metric  $f(R)$  gravity the second derivatives contained in  $\square\phi$  can be eliminated, for the  $\omega = -3/2$  equivalent of Palatini  $f(R)$  gravity this is not possible and, when integrating twice to solve the field equations, the solution for the metric will depend on  $\phi$ , not only on an integral of  $\phi$  (for example through the usual Green function integral). Then, the metric will be very sensitive to variations in  $\phi$  that are not smoothed out, or averaged, by an integral. This extreme sensitivity, contrary to ordinary situations, may spoil the continuous dependence on initial conditions. Physically, this is the situation encountered in modelling stars in Palatini  $f(R)$  gravity. In Ref. [37] it is found that the dependence of the metric on

higher order derivatives of the matter fields makes it so sensitive to small variations that it is impossible to build even a model of a polytropic star, while a polytropic gas is a perfectly reasonable form of matter even in a Newtonian star. It is auspicious that this dependence on higher order derivatives of the matter fields be avoided in future theories of gravity, for example, by including higher order terms in  $R_{ab}R^{ab}$  in the action.

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